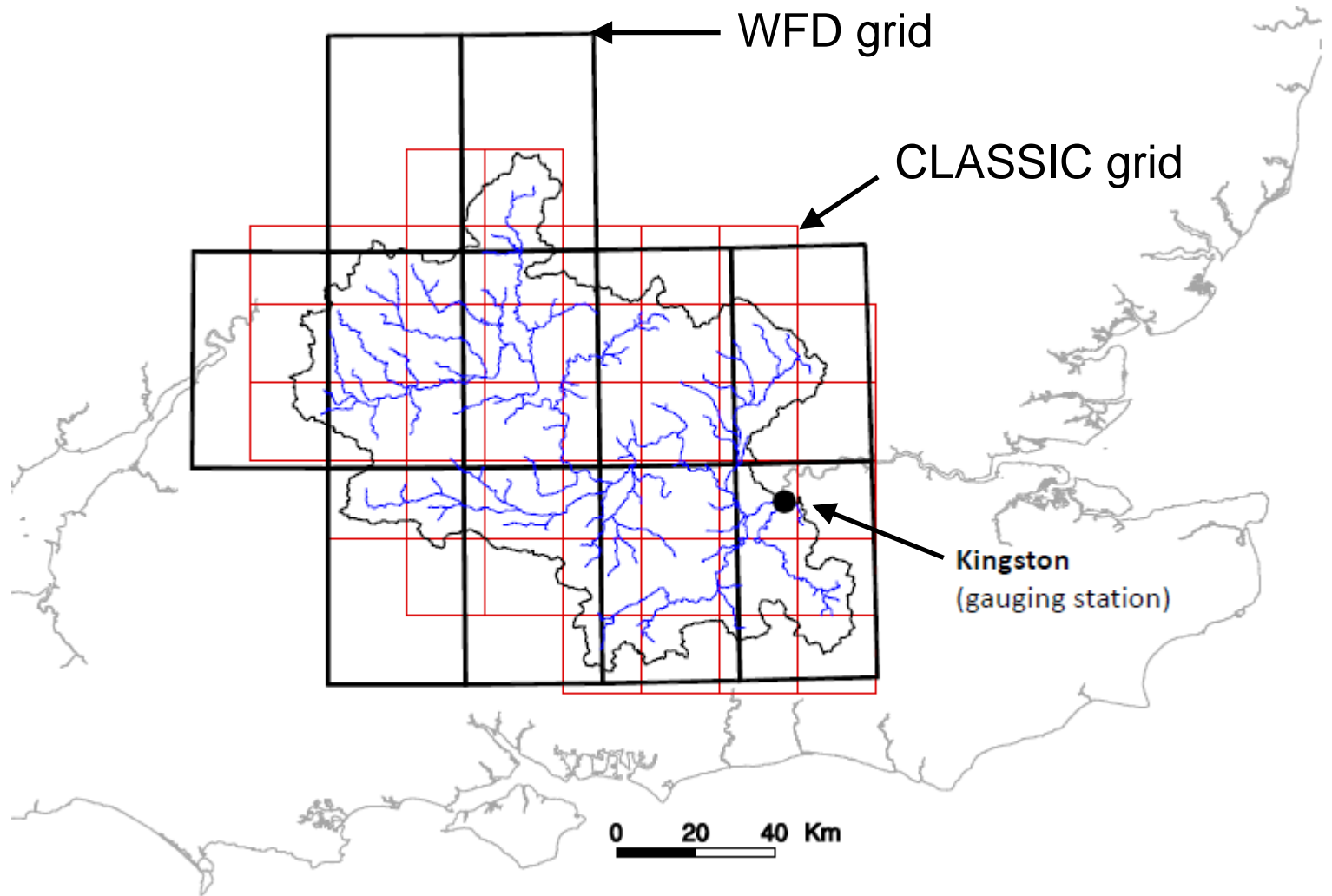


# A spectral approach to assessing JULES performance: case study of late twentieth century discharge from the Thames Basin

Graham Weedon(1), Christel Prudhomme(2), Sue Crooks(2),  
Richard Essery(2), Sonja Folwell(2) and Martin Best(1)

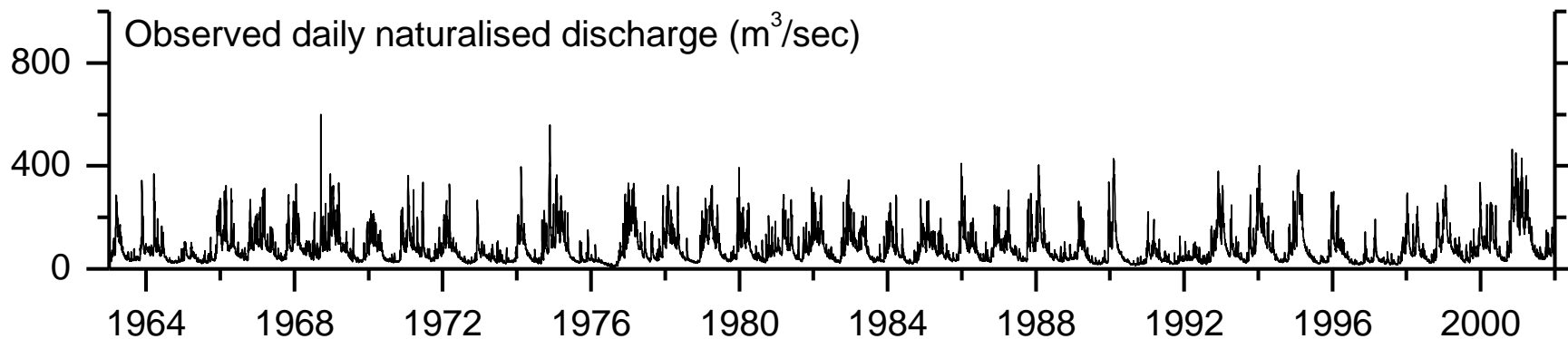
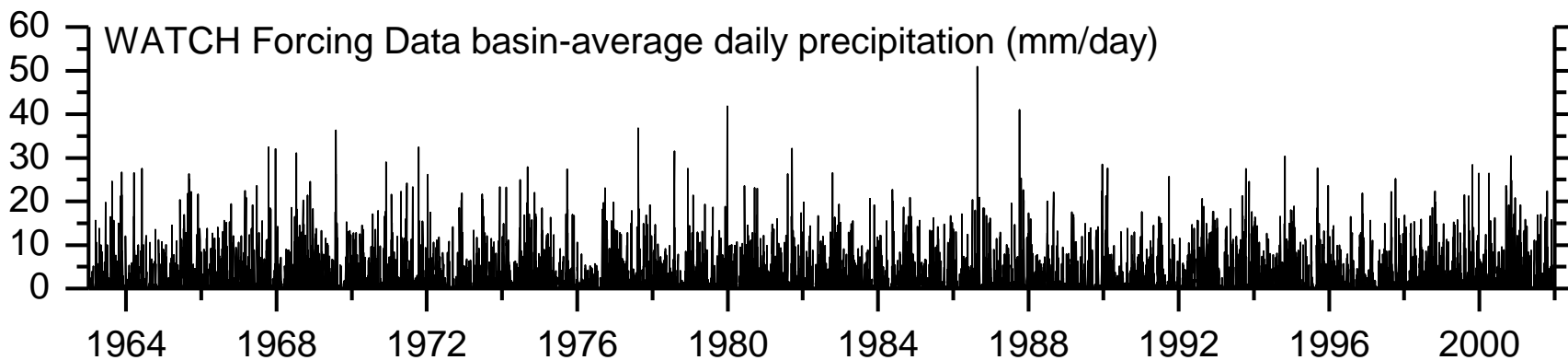
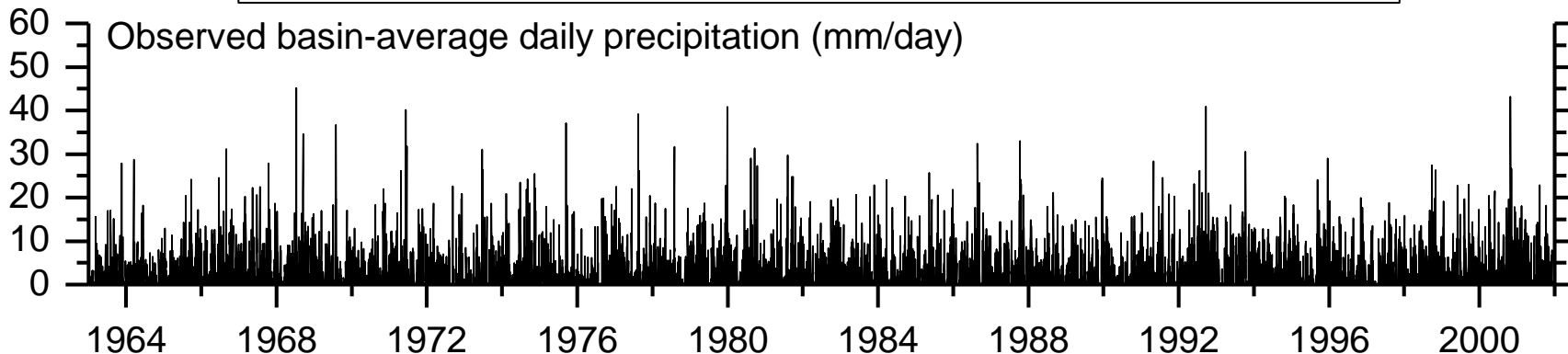
1 = Met Office, UK    2 = CEH Wallingford, UK

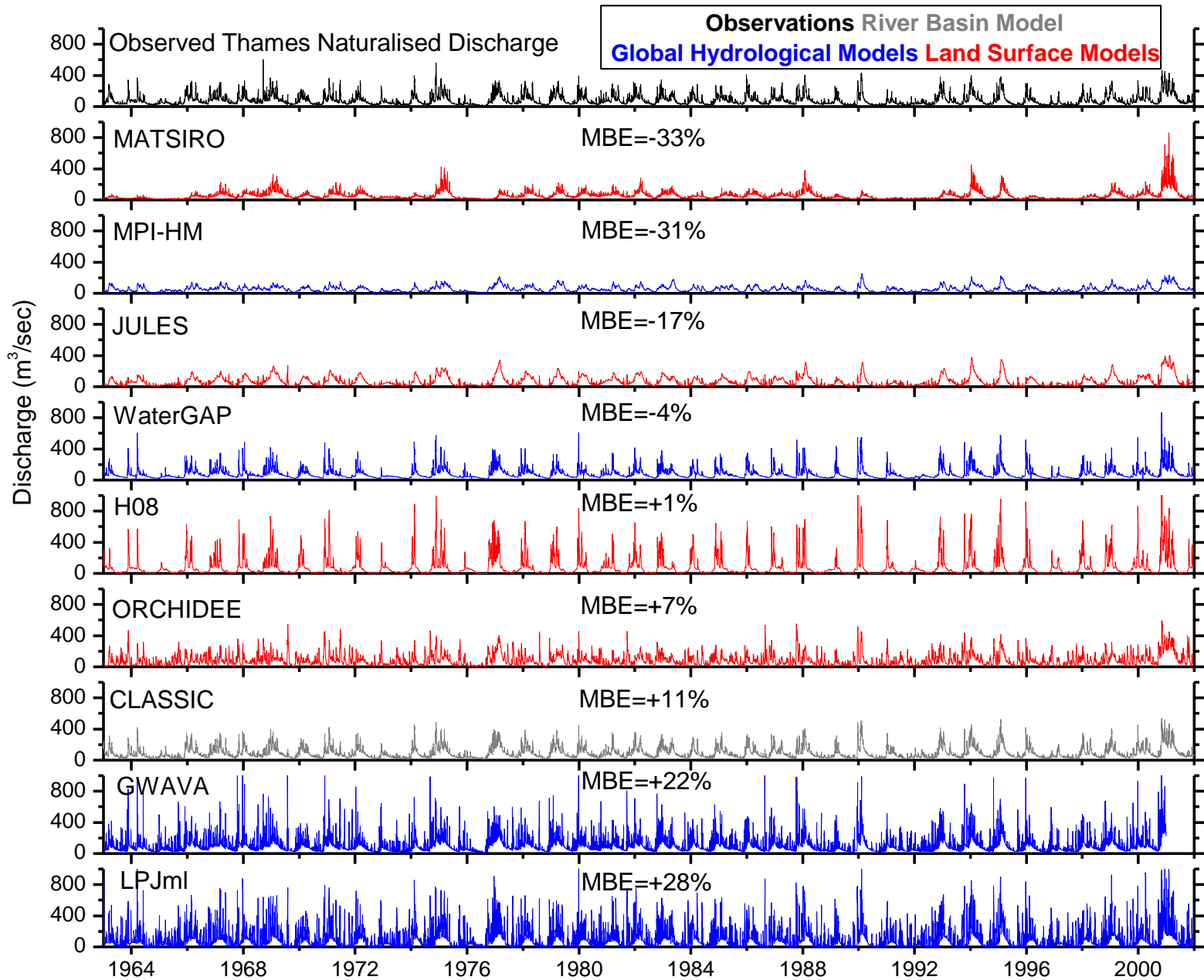


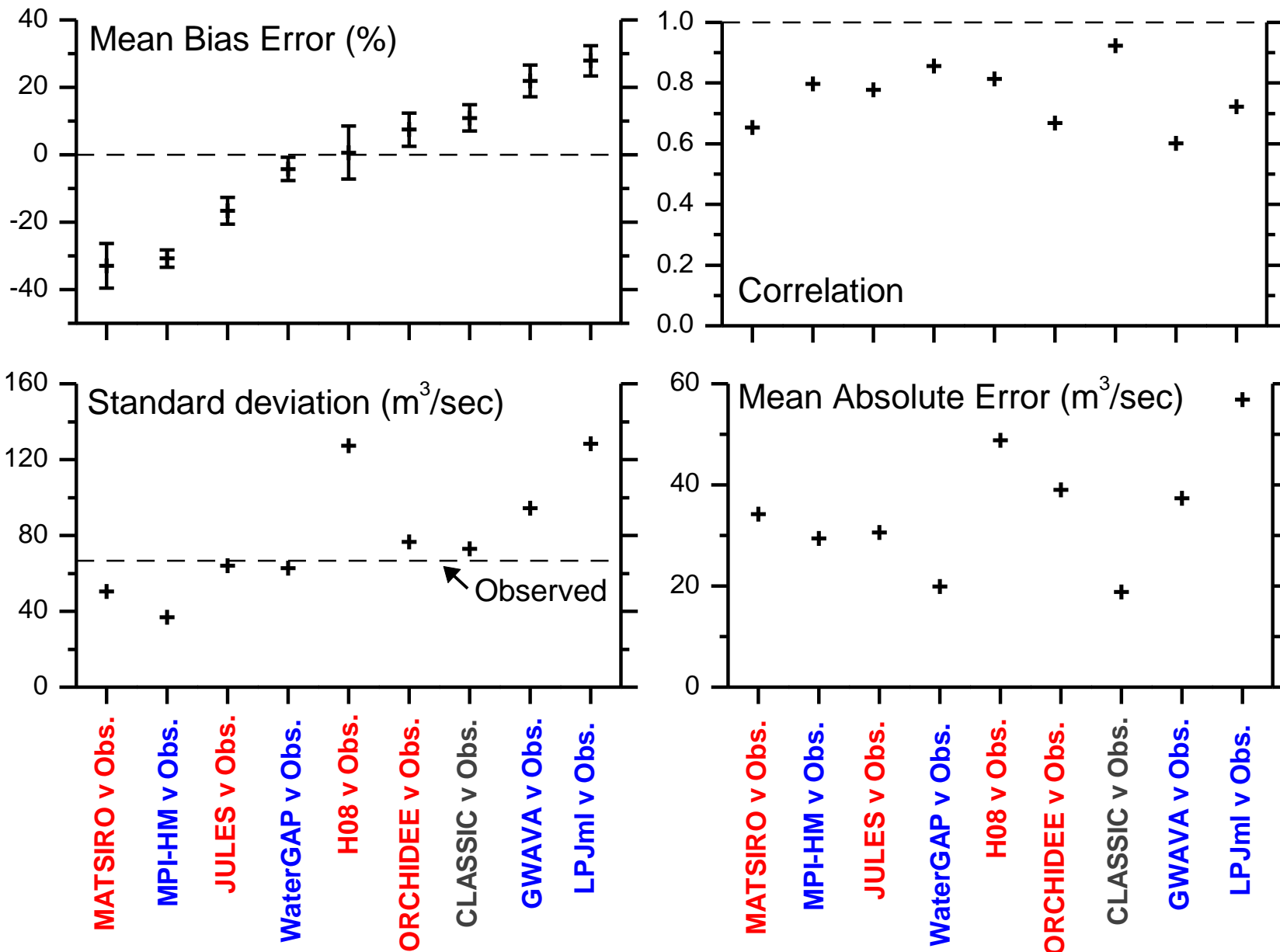
# Modelling the Thames Basin

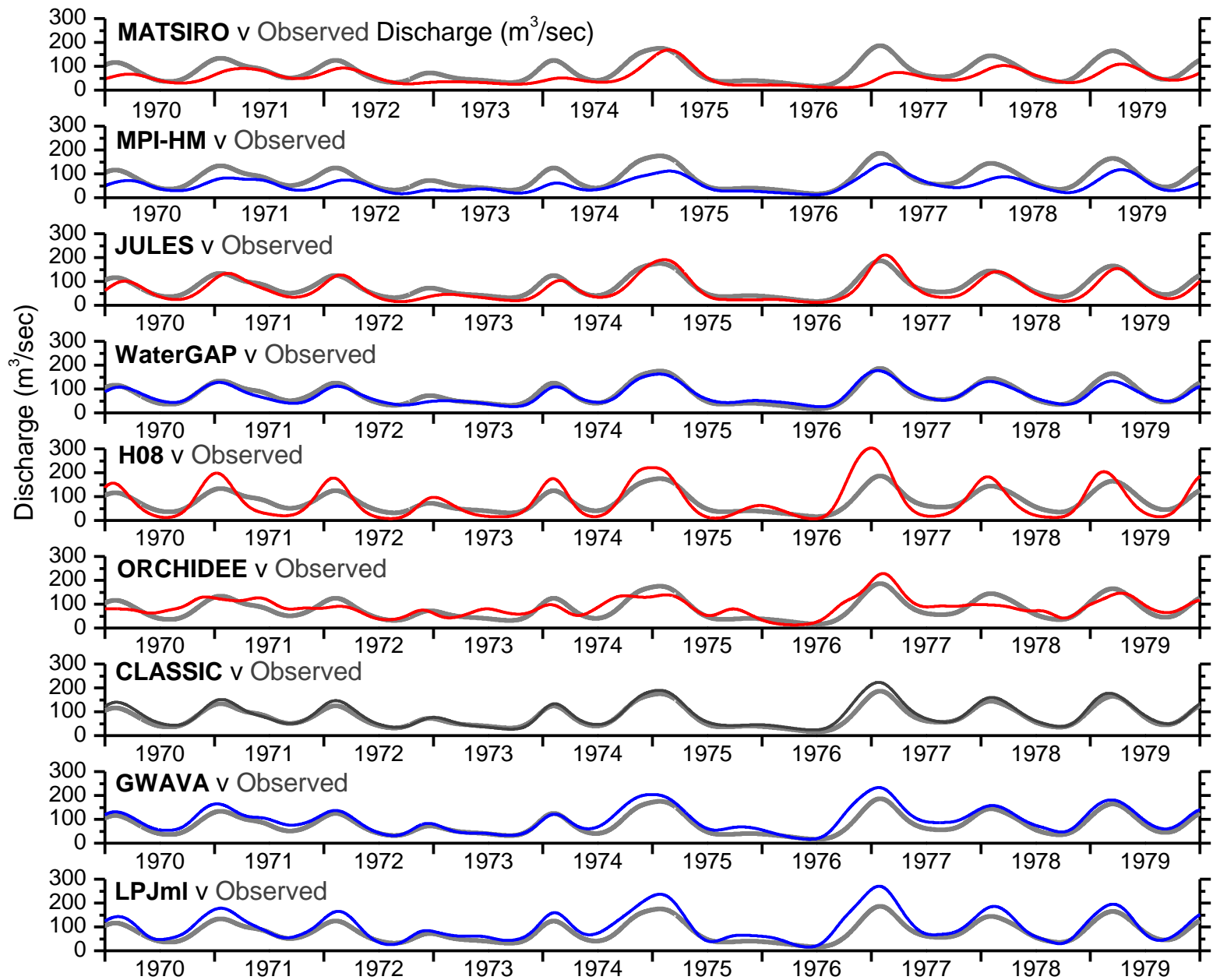
Fig 1 of Crooks, 2011 WATCH Technical Report 53

Precip. Observed	Mean = 1.97 +/-0.06mm	STDev = 3.68mm
Precip. WFD	Mean = 1.99 +/-0.06mm	STDev = 3.79mm
Correlation (Pearson's r)	= 0.635 (P<0.001)	MAE = 1.57mm

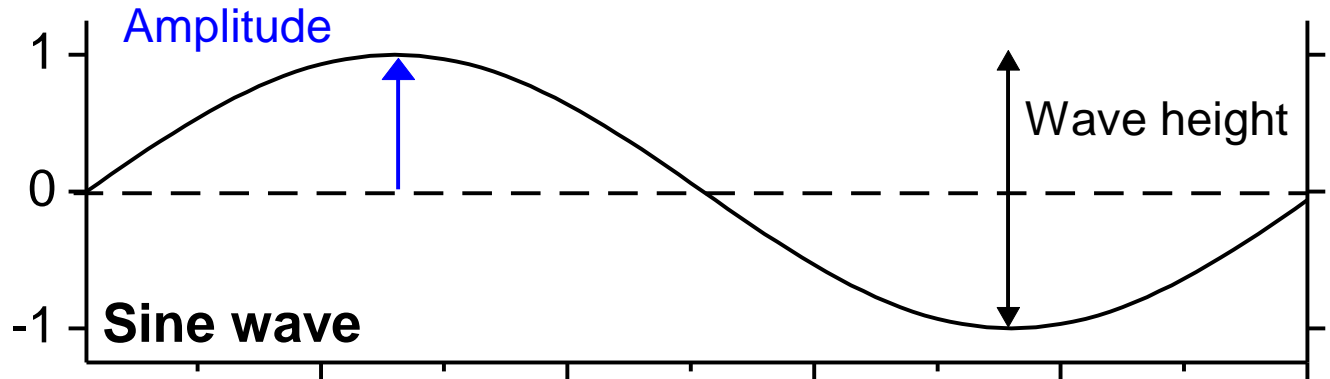








Three parameters describe sinusoidal oscillations:  
**Amplitude**, **Period** and **Phase**



Phase angle

(Degrees) → 0°

90°

180°

270°

360°/0°

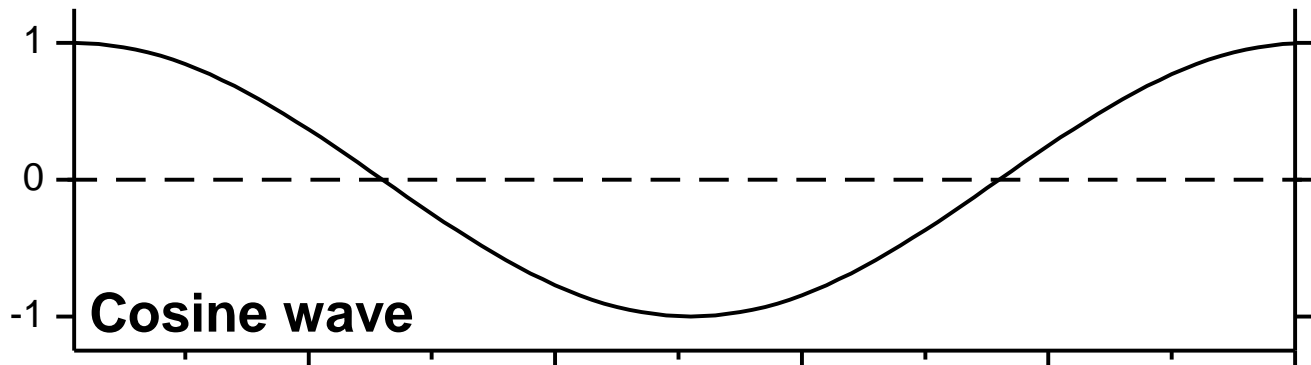
(Radians) → 0

$\pi/2$

$\pi$

$3/2\pi$

$2\pi$



Period (Frequency = 1 / Period)

Time →

← Wavelength (Wave number = 1/wavelength): Distance units →

# Fourier's Theorem:

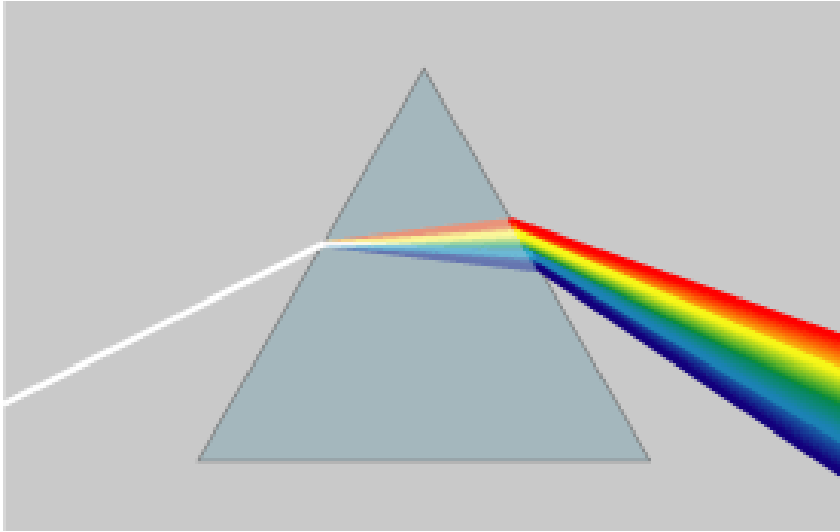
Any time series\* can be represented as the sum of sine and cosine components having the appropriate amplitudes.

\* **Caveat:** The time series must include oscillations, but exclude infinite values.

**Methodology:** Apply the Fourier Transform to the time series.



# Fourier Transform: analogy with optics

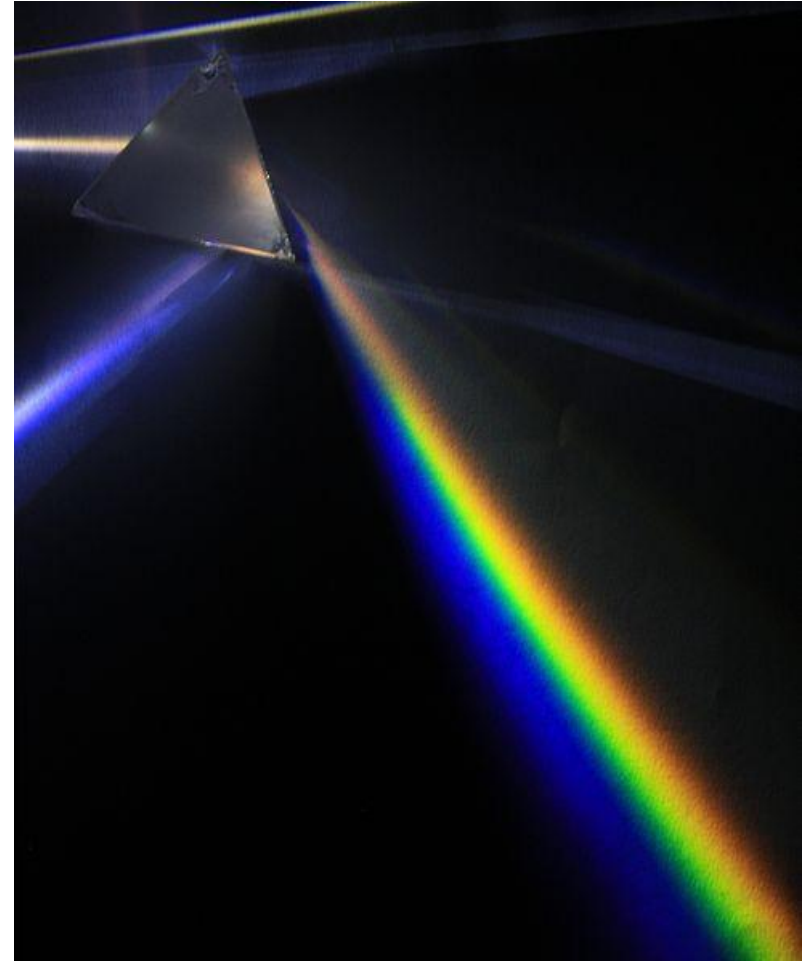


## Theory:

White light is separated into frequency components using a glass prism. Different wavelengths are refracted by different amounts. Red light has a longer wavelength than blue light.

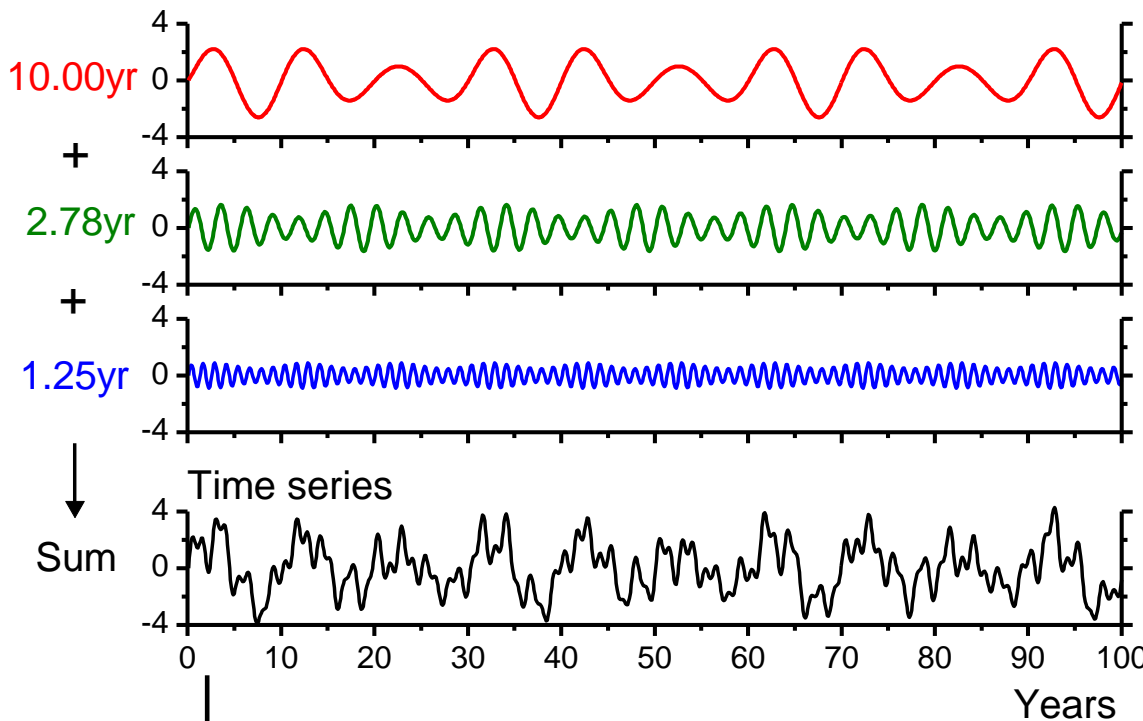
Image: [Merganser.math.gvsu.edu](http://Merganser.math.gvsu.edu)

© Crown copyright Met Office



White light from a Mercury vapour lamp passed through a flint glass prism.

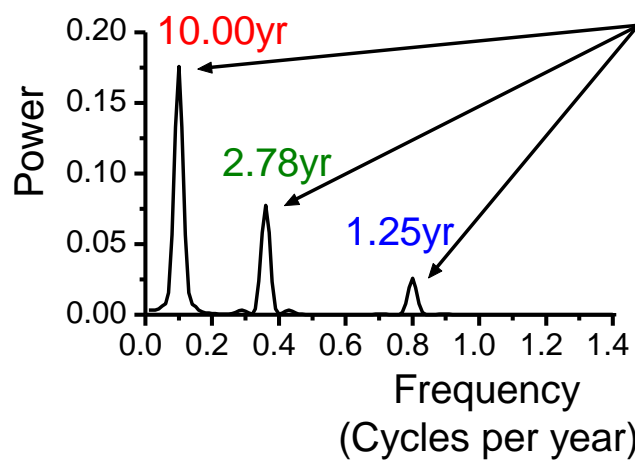
Image: [en.wikipedia.org/Visible\\_spectrum](http://en.wikipedia.org/Visible_spectrum)



Spectral analysis of "time series" involves the use of amplitude and frequency (= 1/period) only.

The power spectrum plots the **average** squared amplitude (= power or variance).

Discrete Fourier Transform



The spectrum shows regular cycles emerging as spectral peaks.

**INPUT**

Observed Precipitation



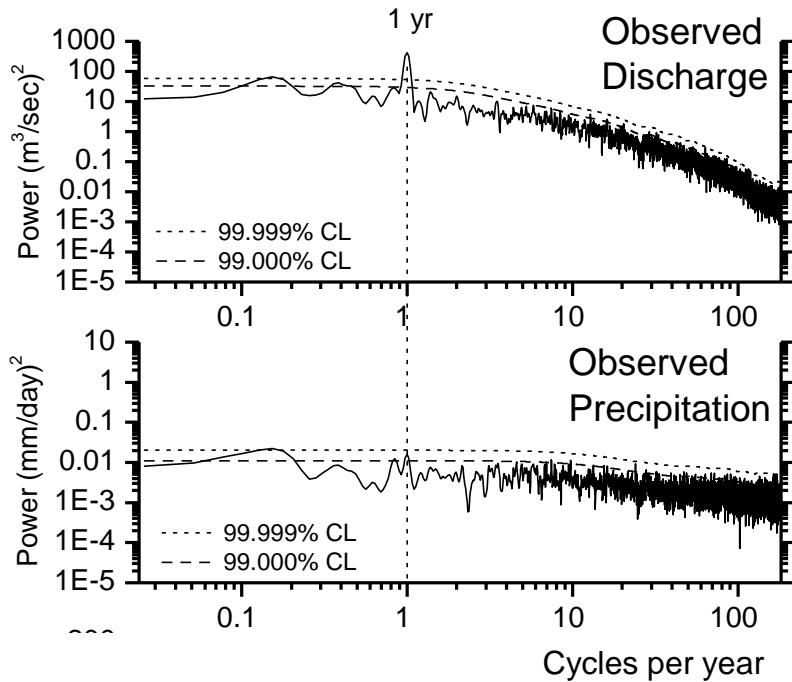
**SYSTEM**

Thames Basin



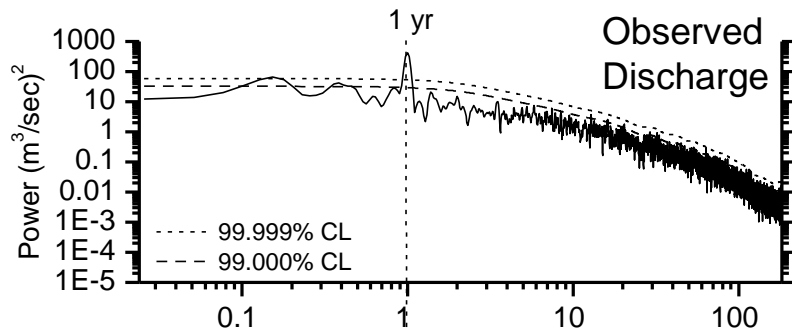
**OUTPUT**

Observed Naturalised  
Discharge

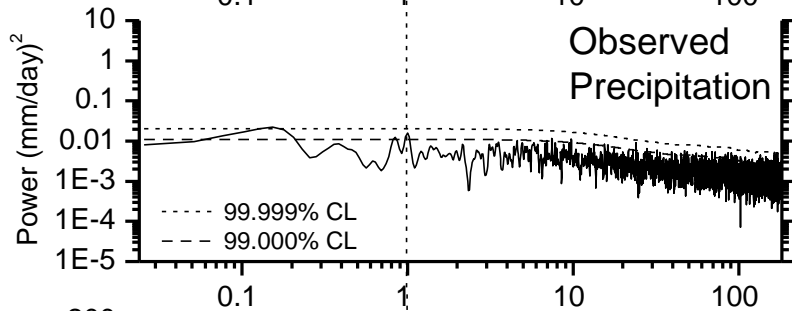


← Shape of Discharge power spectrum implies strong annual cycles plus strong autocorrelation (“memory”).

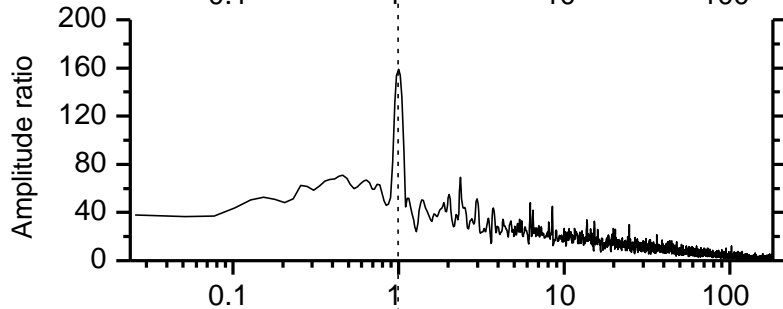
← Shape of Precipitation power spectrum implies weak annual cycles plus weak autocorrelation (“memory”).



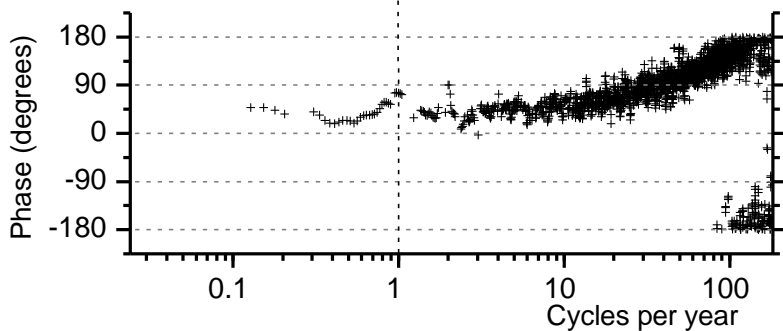
← Power spectrum of output



← Power spectrum of input



← Amplitude-ratio spectrum indicates strong amplification of annual cycle variation.



← Phase (difference) spectrum indicates difference in timing of oscillations of the output series compared to the input.

**INPUT**

WFD Precipitation



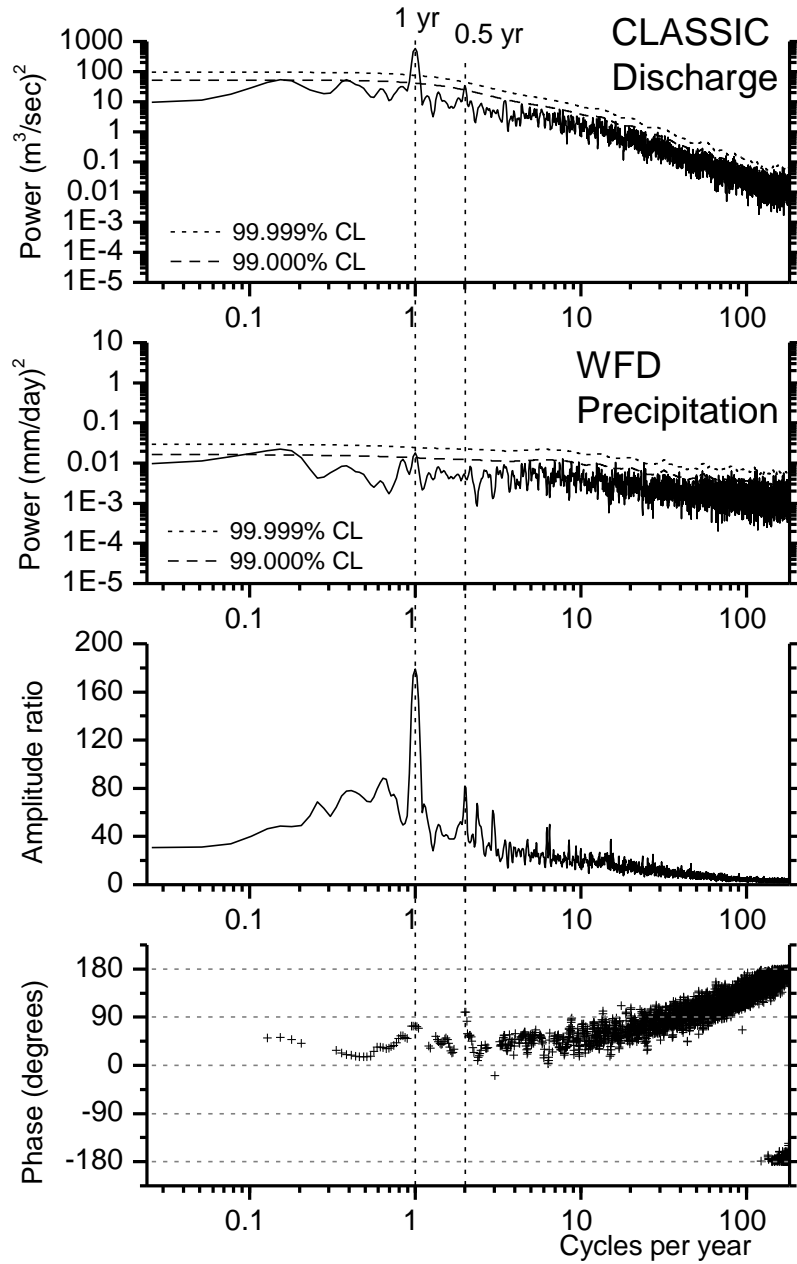
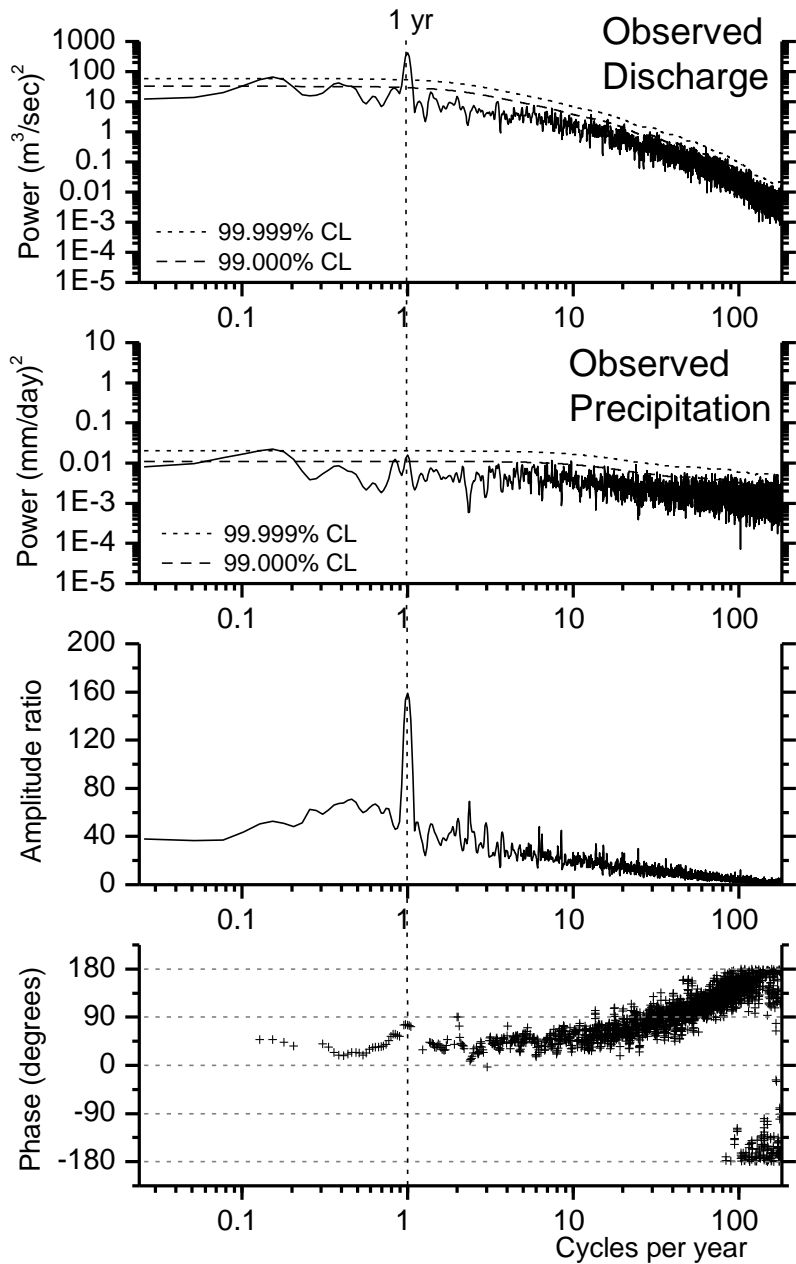
**SYSTEM**

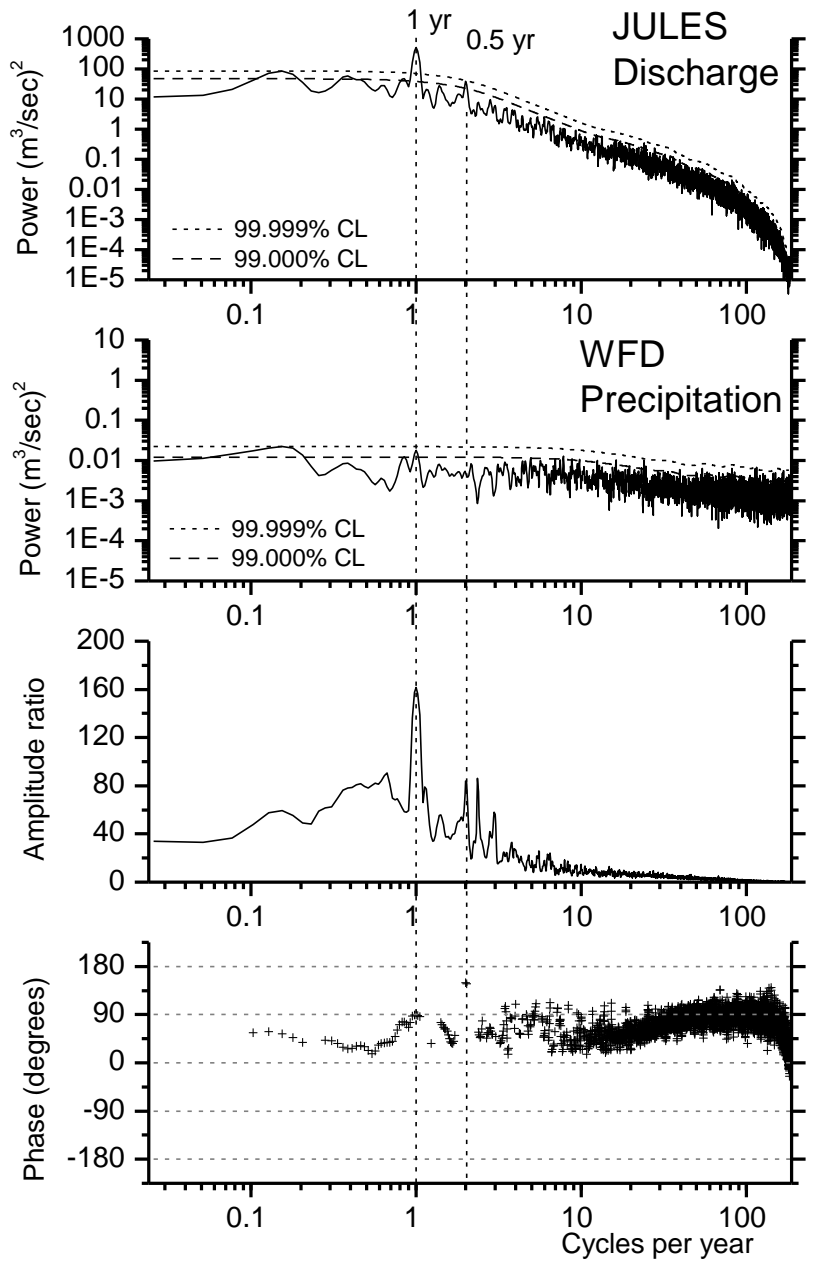
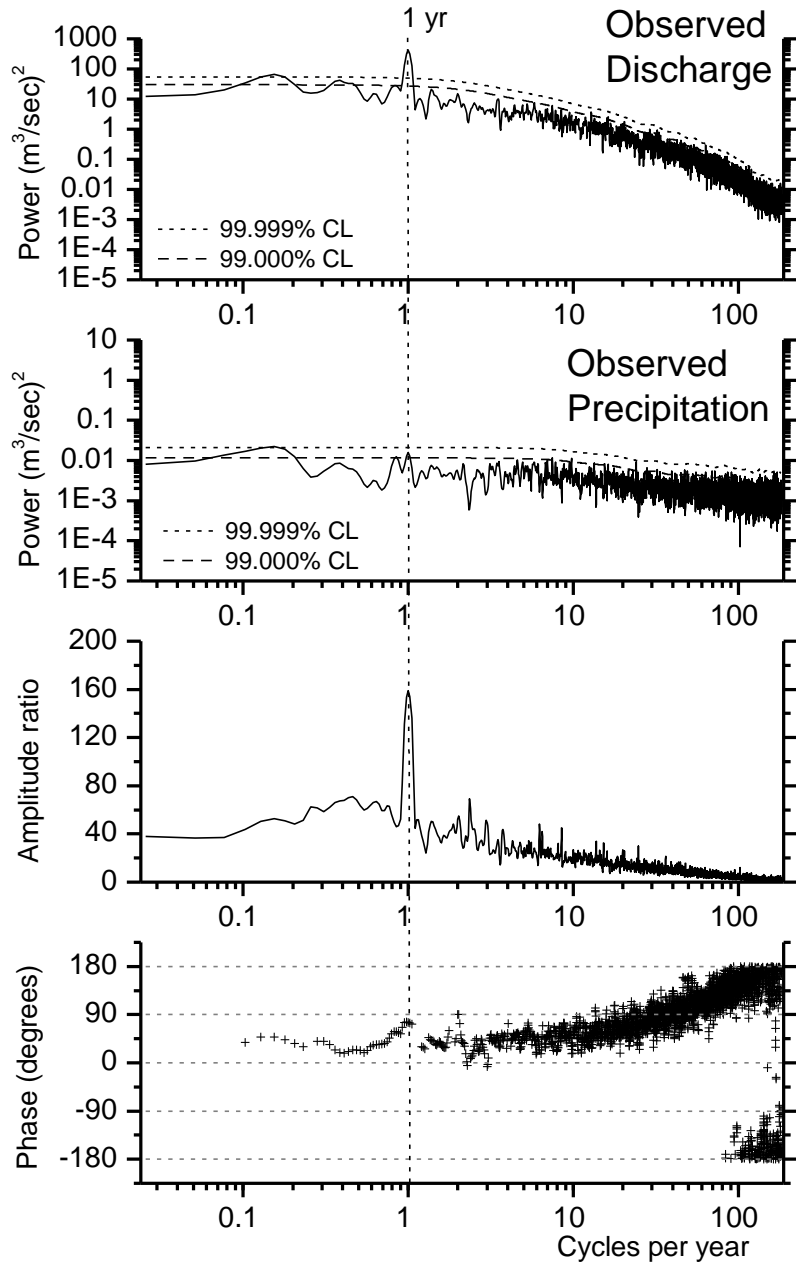
Hydrological model



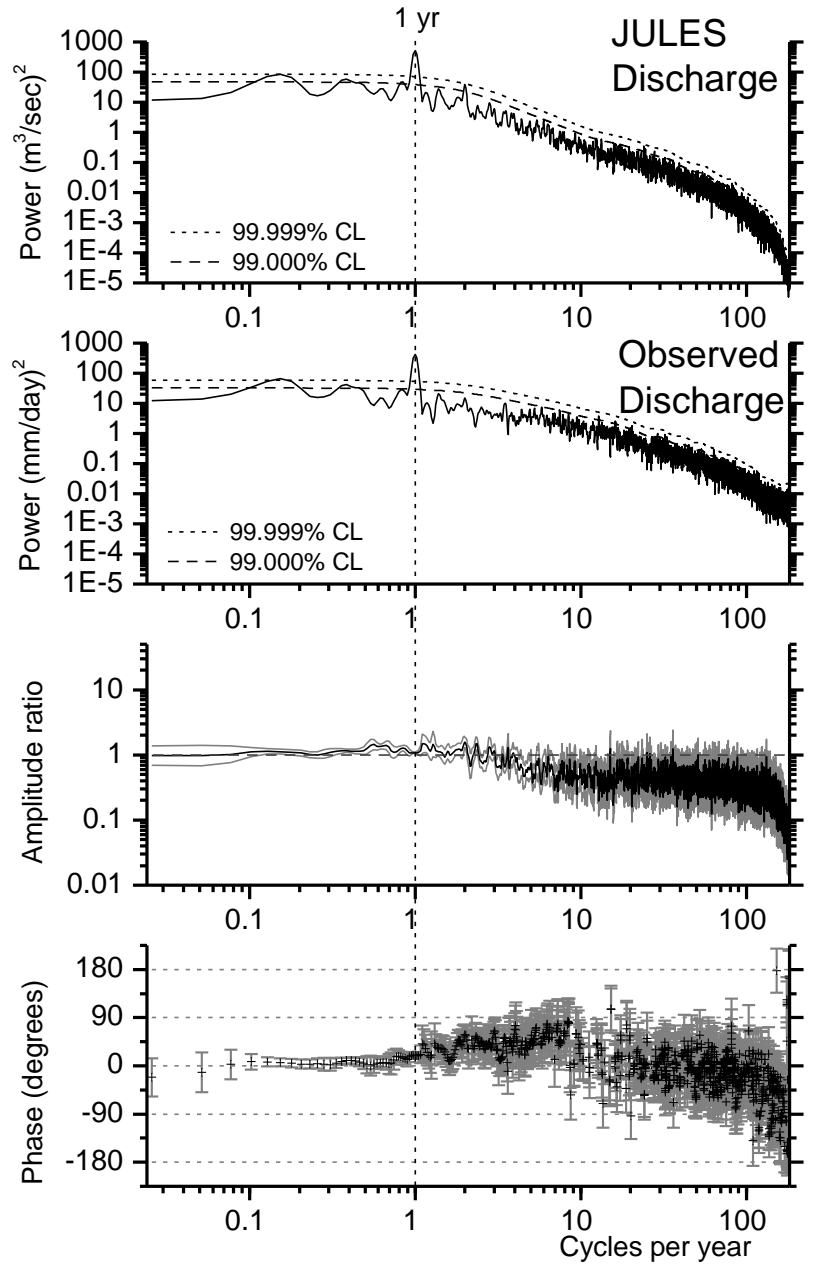
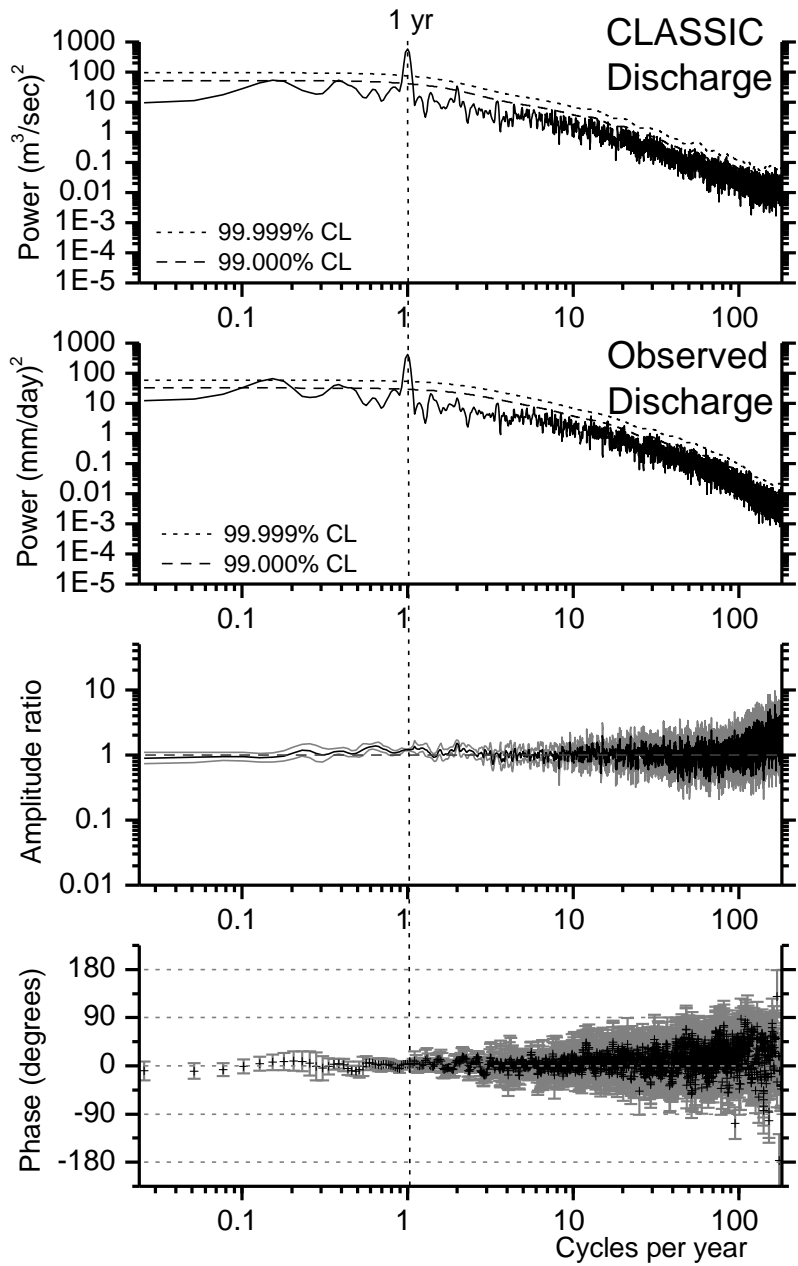
**OUTPUT**

Modelled Discharge

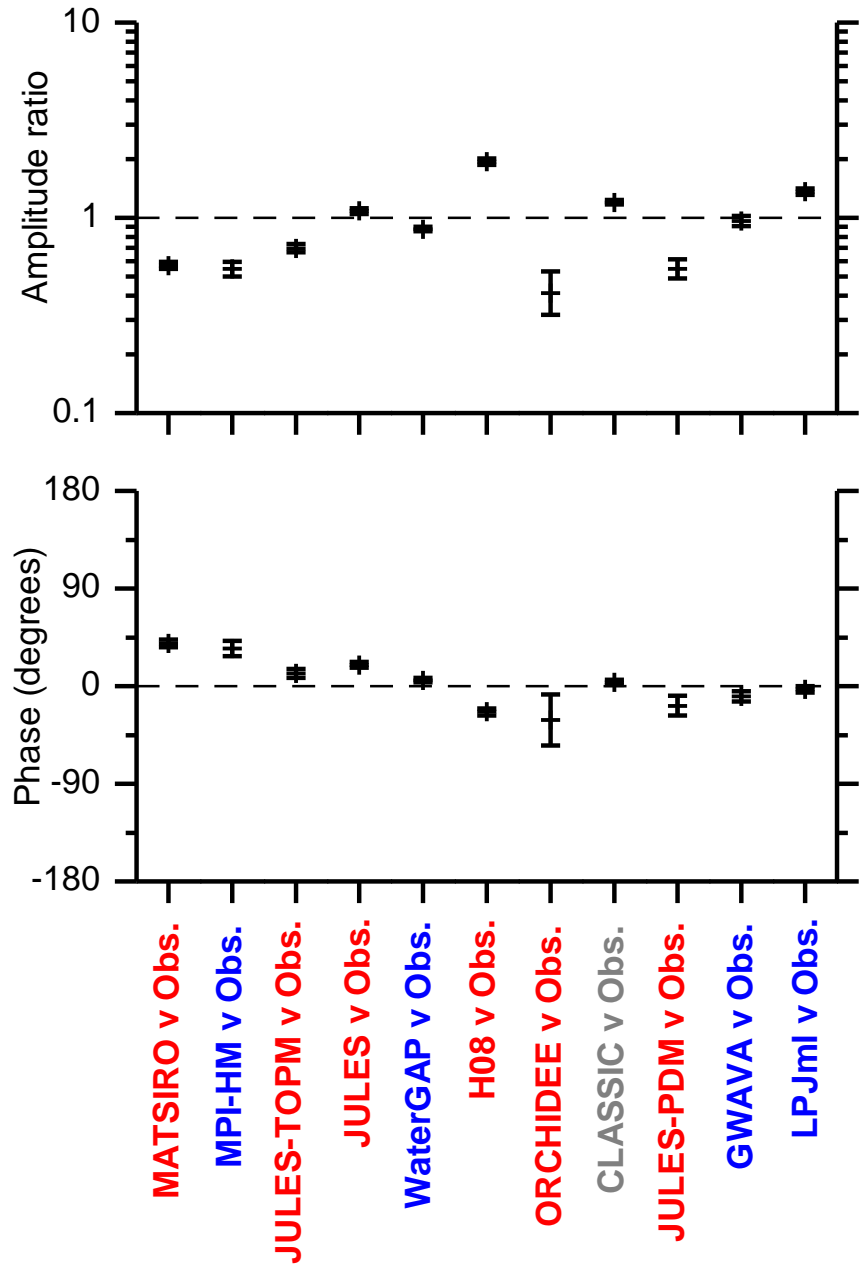


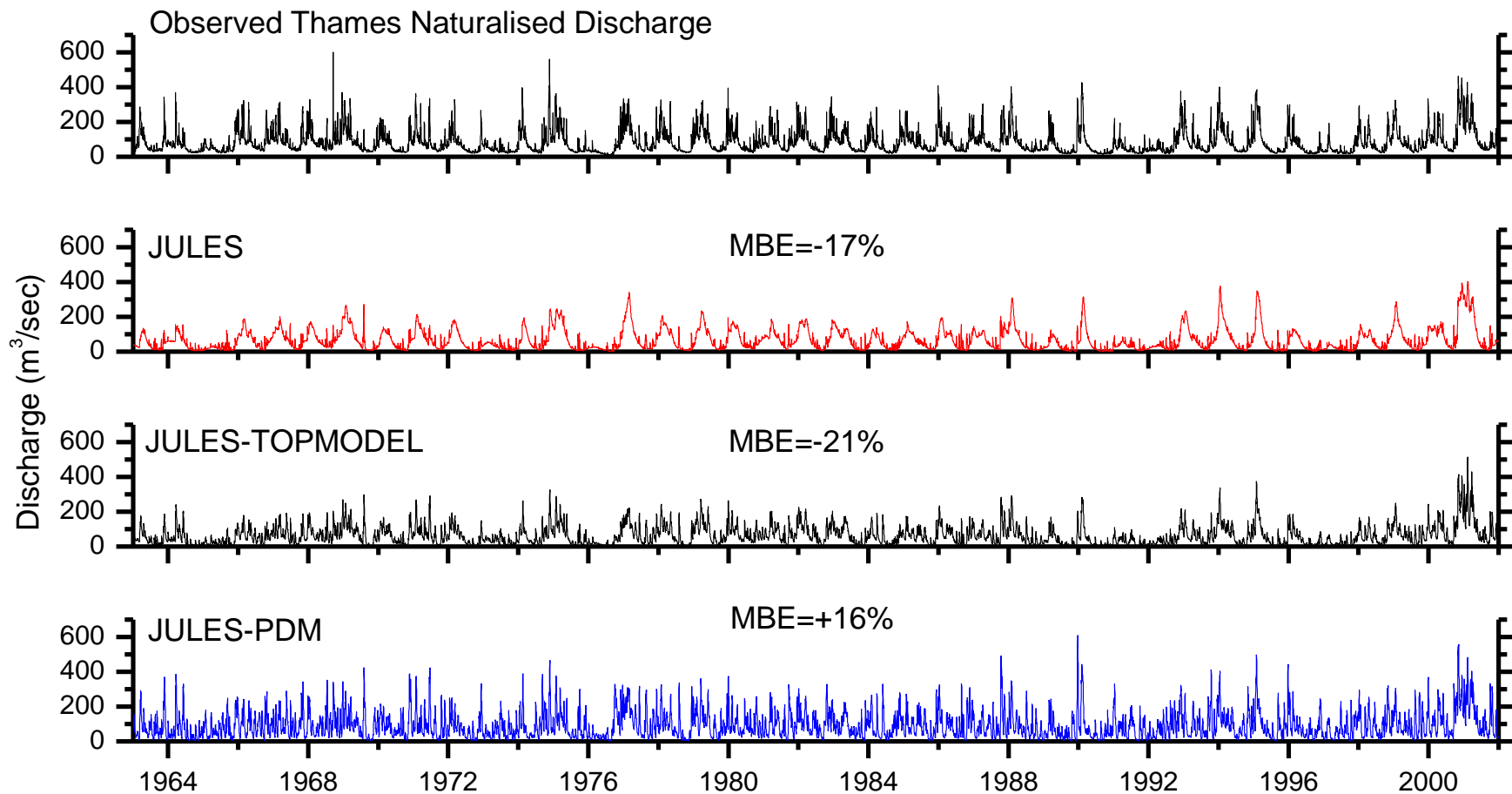


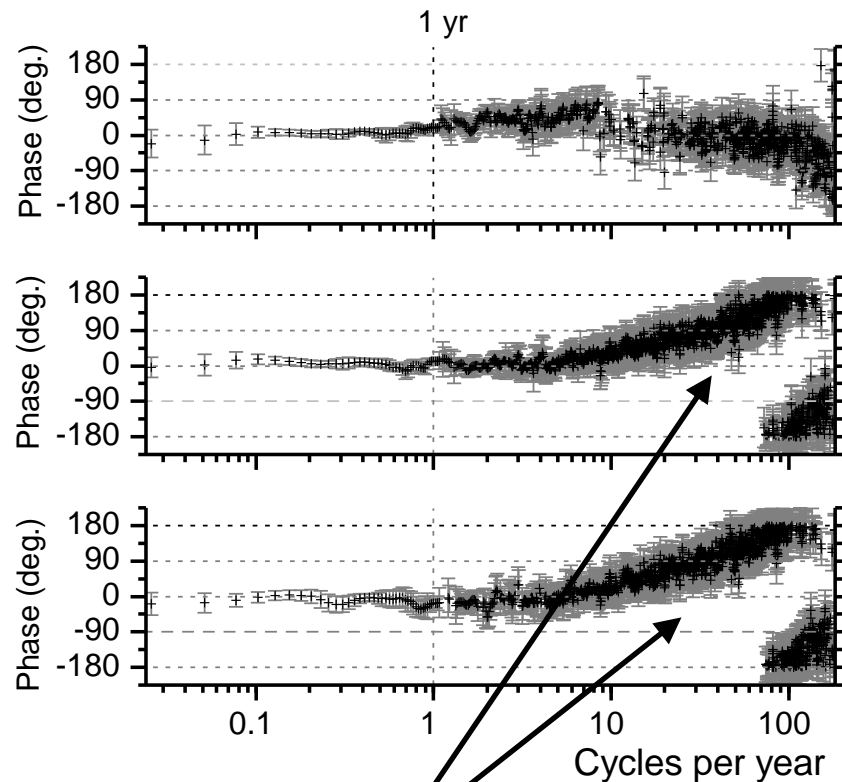
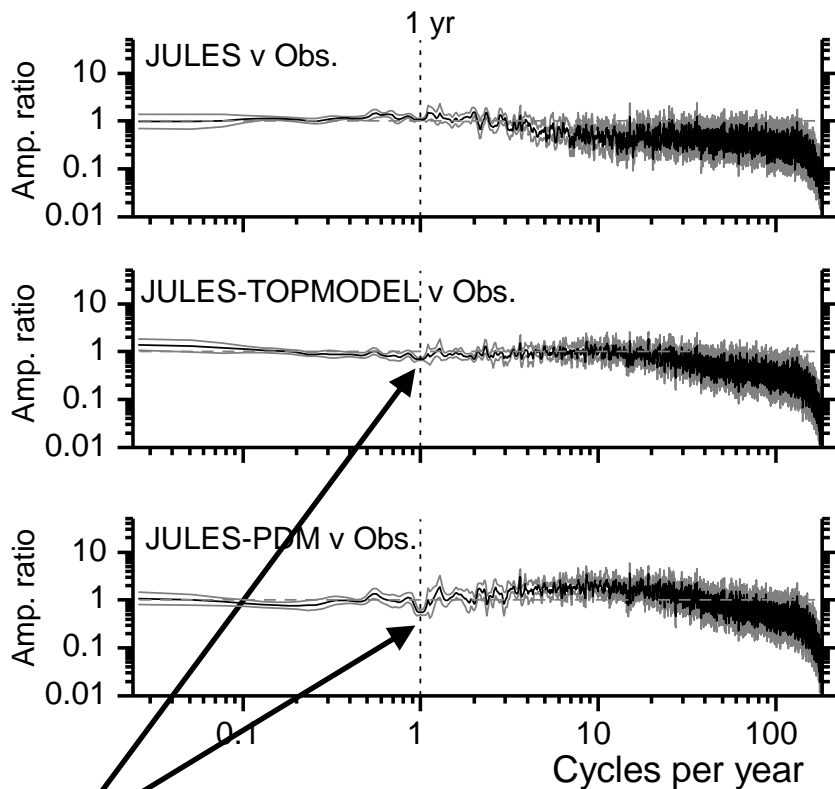




# Model evaluation at the annual scale







JULES-TOPMODEL & JULES-PDM have worse reproduction of the observed annual amplitude of discharge compared to JULES.

JULES-TOPMODEL & JULES-PDM have worse reproduction of the observed high frequency discharge phases compared to JULES.

**INPUT**

WFD Precipitation



**SYSTEM**

Hydrological model

Runoff modelling



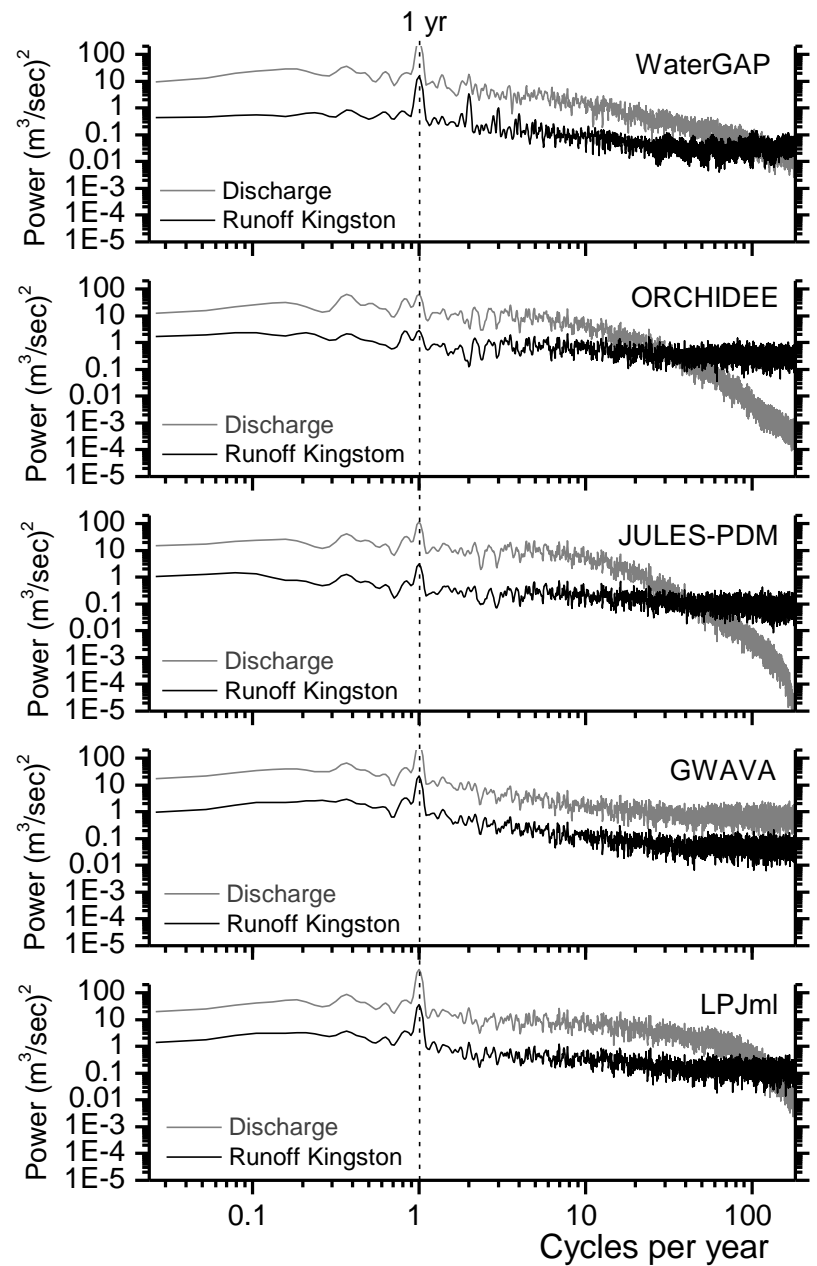
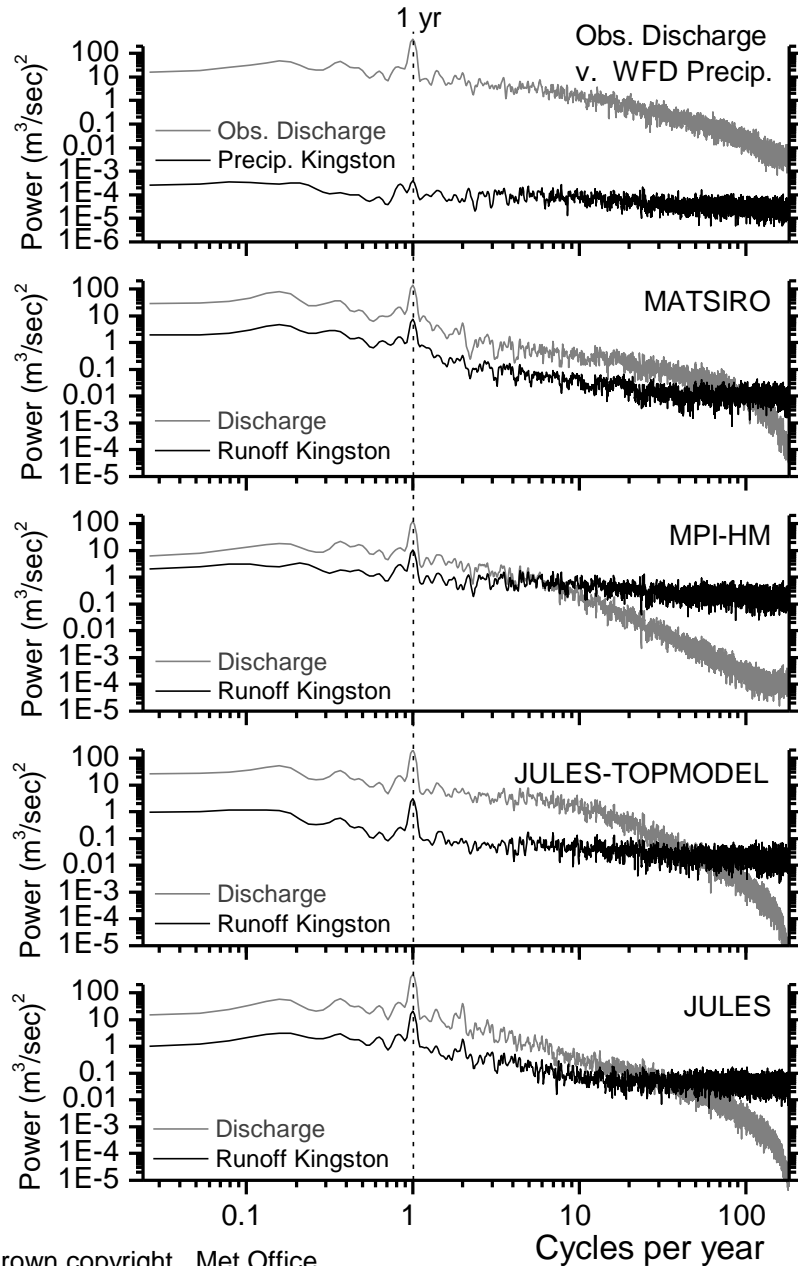
Routing modelling



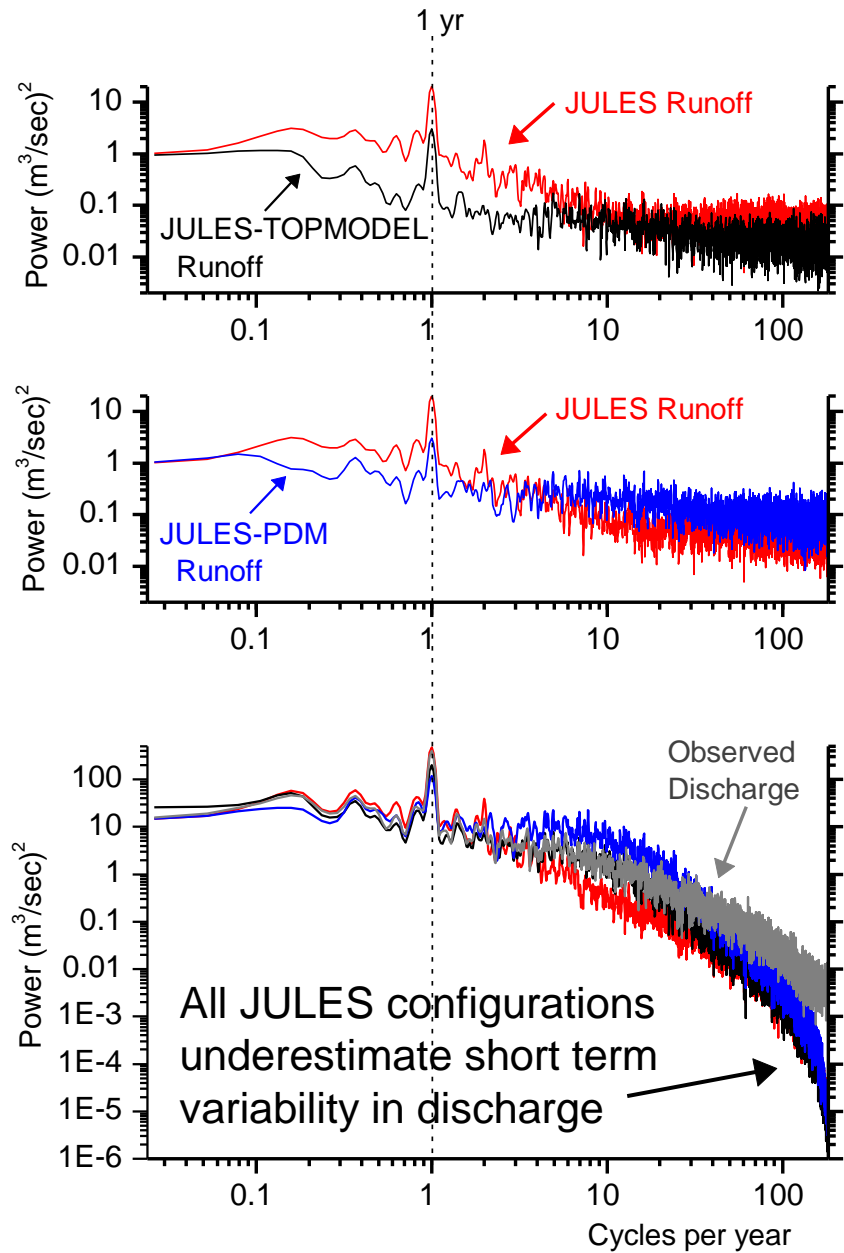
**OUTPUT**

Modelled Discharge

# Power spectra of Kingston grid box **Runoff** v. power spectra of **Discharge**



# Runoff v. Discharge power spectra for JULES



# Simulation of runoff and discharge spectral characteristics

$P(t)$  = Observed daily precipitation

Simulating annual cycle in evaporation

$$R(t) = P(t) + \text{Sine}(1\text{yr})$$

Adding weak autocorrelation  
(Runoff "memory")

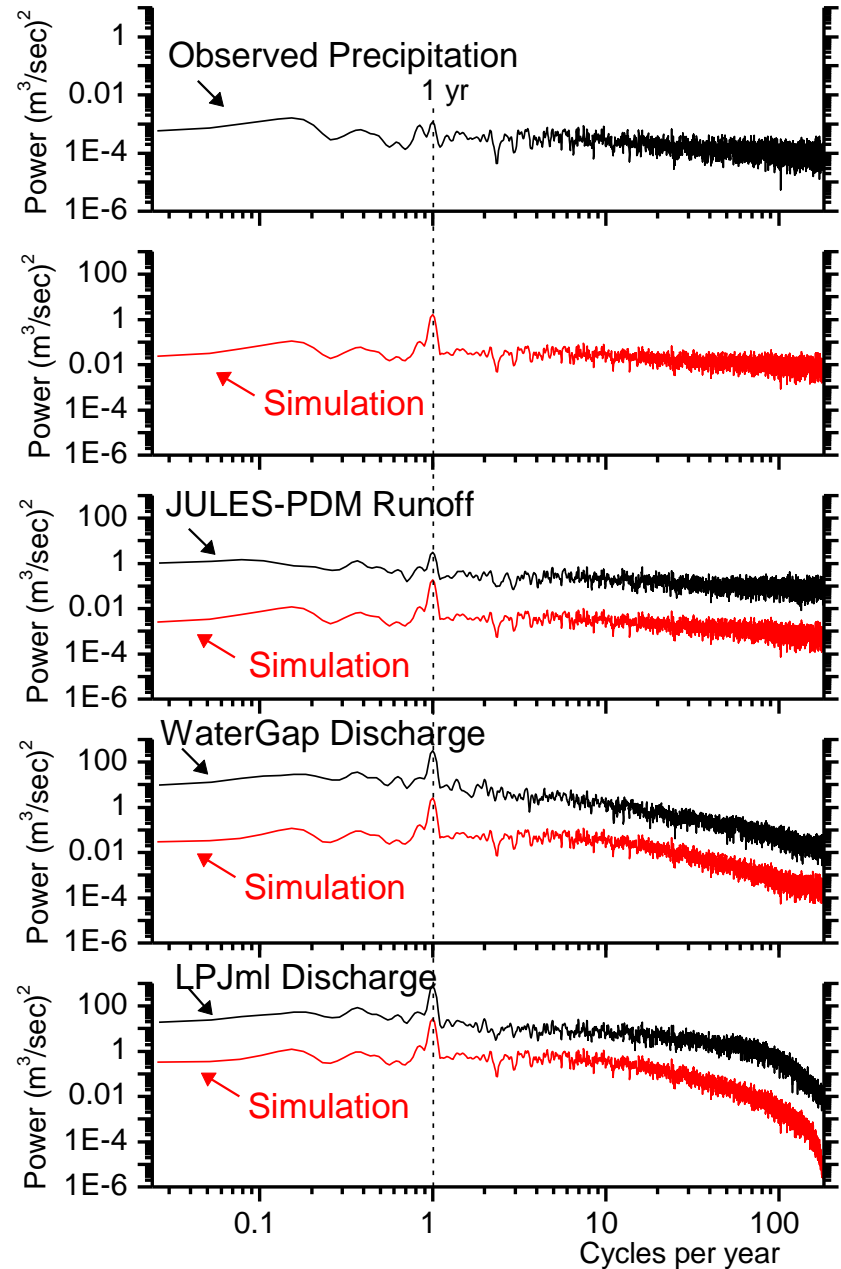
$$R(t) = R(t) + 0.1 * R(t)$$

Adding Strong autocorrelation  
(Routing "memory")

$$D(t) = R(t) + 0.7 * D(t-1)$$

Adding moving-average  
(Routing artifact)

$$D(t) = [D(t) + D(t-1)]/2$$





# Conclusions:

- 1) Cross-spectral analysis provides quantitative assessment of model performance in terms of reproducing observed amplitude and phase at different time scales. Separation of performance into different time scales allows consideration of model performance in terms of different physical processes.
- 2) Although using TOPMODEL or PDM allows some improvement of short-term (weekly) variability in discharge, it also leads to degraded performance in terms of amplitude at annual scales.
- 3) In terms of reproducing discharge variability, provided the overall evaporation is right (assessed via MBE): then modelling of routing is more important than modelling runoff.
- 4) JULES (and other models) introduces a “moving-average artifact” causing suppression of very short-term (few days) variability when simulating discharge related to representation of routing (caused by grid box to grid box transfer of water).