A spectral approach to assessing JULES performance: case study of late twentieth century discharge from the Thames Basin

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Modelling the Thames Basin

Fig 1 of Crooks, 2011 WATCH Technical Report 53
Precip. Observed  Mean = 1.97 +/-0.06mm  STDev = 3.68mm
Precip. WFD     Mean = 1.99 +/-0.06mm  STDev = 3.79mm
Correlation (Pearson’s r) = 0.635 (P<0.001)       MAE = 1.57mm

Observed basin-average daily precipitation (mm/day)

WATCH Forcing Data basin-average daily precipitation (mm/day)

Observed daily naturalised discharge (m$^3$/sec)
Three parameters describe sinusoidal oscillations: Amplitude, Period and Phase.

- **Amplitude**: The maximum value of the oscillation.
- **Period**: The time for one complete cycle.
- **Phase**: The position of the oscillation at a given time.

The wave height is given by the amplitude. The wavelength (Wave number = 1/wavelength): Distance units) is the distance between two consecutive peaks or troughs.

The wave equation for a sine wave is given by $y(t) = A \sin(2\pi f t + \phi)$, where $A$ is the amplitude, $f$ is the frequency, $t$ is time, and $\phi$ is the phase. For a cosine wave, the equation is $y(t) = A \cos(2\pi f t + \phi)$.
Fourier’s Theorem:
Any time series* can be represented as the sum of sine and cosine components having the appropriate amplitudes.

*Caveat: The time series must include oscillations, but exclude infinite values.

Methodology: Apply the Fourier Transform to the time series.
Fourier Transform: analogy with optics

**Theory:**
White light is separated into frequency components using a glass prism. Different wavelengths are refracted by different amounts. Red light has a longer wavelength than blue light.

Image: Merganser.math.gvsu.edu

White light from a Mercury vapour lamp passed through a flint glass prism.

Image: en.wikipedia.org/Visible_spectrum
Spectral analysis of "time series" involves the use of amplitude and frequency (= 1/period) only.

The power spectrum plots the average squared amplitude (= power or variance).

The spectrum shows regular cycles emerging as spectral peaks.

The power spectrum plots the average squared amplitude (= power or variance).
INPUT
Observed Precipitation

SYSTEM
Thames Basin

OUTPUT
Observed Naturalised Discharge
Shape of Discharge power spectrum implies strong annual cycles plus strong autocorrelation ("memory").

Shape of Precipitation power spectrum implies weak annual cycles plus weak autocorrelation ("memory").
Amplitude-ratio spectrum indicates strong amplification of annual cycle variation.

Phase (difference) spectrum indicates difference in timing of oscillations of the output series compared to the input.
INPUT
WFD Precipitation

SYSTEM
Hydrological model

OUTPUT
Modelled Discharge
Model evaluation at the annual scale
Observed Thames Naturalised Discharge

JULES

JULES-TOPMODEL

JULES-PDM

Discharge (m\(^3/\)sec)

MBE=-17%

MBE=-21%

MBE=+16%
JULES-TOPMODEL & JULES-PDM have worse reproduction of the observed annual amplitude of discharge compared to JULES.

JULES-TOPMODEL & JULES-PDM have worse reproduction of the observed high frequency discharge phases compared to JULES.
INPUT
WFD Precipitation

SYSTEM
Hydrological model
  Runoff modelling
  Routing modelling

OUTPUT
Modelled Discharge
Power spectra of Kingston grid box Runoff v. power spectra of Discharge

- Obs. Discharge v. WFD Precip.
- WaterGAP
- MATSIRO
- MPI-HM
- JULES-TOPMODEL
- JULES-PDM
- ORCHIDEE
- GWAVA
- LPJml

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All JULES configurations underestimate short term variability in discharge.
Simulation of runoff and discharge spectral characteristics

\[ P(t) = \text{Observed daily precipitation} \]

Simulating annual cycle in evaporation
\[ R(t) = P(t) + \text{Sine}(1\text{yr}) \]

Adding weak autocorrelation
(Runoff "memory")
\[ R(t) = R(t) + 0.1 \times R(t) \]

Adding Strong autocorrelation
(Routing "memory")
\[ D(t) = R(t) + 0.7 \times R(t-1) \]

Adding moving-average
(Routing artifact)
\[ D(t) = \frac{[D(t) + D(t-1)]}{2} \]

\[ D(t) = \frac{[D(t) + D(t-1)]}{2} \]
Conclusions:

1) Cross-spectral analysis provides quantitative assessment of model performance in terms of reproducing observed amplitude and phase at different time scales. Separation of performance into different time scales allows consideration of model performance in terms of different physical processes.

2) Although using TOPMODEL or PDM allows some improvement of short-term (weekly) variability in discharge, it also leads to degraded performance in terms of amplitude at annual scales.

3) In terms of reproducing discharge variability, provided the overall evaporation is right (assessed via MBE): then modelling of routing is more important than modelling runoff.

4) JULES (and other models) introduces a “moving-average artifact” causing suppression of very short-term (few days) variability when simulating discharge related to representation of routing (caused by grid box to grid box transfer of water).